A Primer on Directional Similarity

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Introduction

Because my paper on directional similarity[1] is not readily available to a non-specialist audience, and because the presentation there could stand a bit of correcting, I have decided to write up this primer, which should serve to introduce the concept of directional similarity and the mathematics which comes along with it to a general audience. Because the target audience is not some group of specialists (physicists, mathematicians, people who can balance swords on their chin, etc.), just about everyone should feel at home in reading this paper.

The Basic Concept

Nearly everyone has some familiarity with directional similarity, although they may not call it by that name, or even have any name for it as yet. You might make use of a sense of directional similarity when you walk or drive around, or when you read a compass, or when you say what time it is after looking at the hands of a clock. Directional similarity comes into play when something is not exactly lined up with a given direction, but you have a sense that it is 'close enough.'

When we drive around, it is impossible (usually) for us to drive straight to our destination: the roads are not laid out that way, and so we have to pick a route which, by a series of compromises, gets us to our destination. We can pick such a route because we have a sense of directional similarity.

I once had a wind-up toy robot. It wasn't much of a robot; it just looked like one, but if you wound it up, it marched forward. That robot had no sense of directional similarity. If it encountered an obstruction, it just kept trying to march forward. The end result was that its feet kept moving, but it didn't get anywhere. We can be thankful that we have better sense than that, and that we can recognize that, often, in order to get to where you want to go, sometimes you have to go in a different direction than the one that leads straight to your destination.

[1] Newell, J. "Probabilities Based on Directional Similarity" *The Mathematical Scientist* **39** (2014)

The trick to getting where you want to go, in an effective and efficient way, is to select those paths which are most similar to the ideal path when the ideal path itself is not available. But how do you judge which alternate path is most similar to the ideal path?

Sometimes it's rather obvious: you're walking along and then there's something in your way. Let's say there's a hole in the ground. So, naturally, you make a minimal change of direction and walk around the hole—it'd be silly to walk around the block, or go back home and start all over again. Other times the situation is not that clear: your normal route to work is jammed up, so you could go through the neighborhoods or you could go through the park; the neighborhood route is more direct, but there are lots of traffic stops, the park has fewer stops, but the road winds around and so it is much longer. Figuring out which path is best might just take some research.

If you don't have time for research, you'll just have to take your best guess, and everyone else will too. So now, some will stay on the original path, some will go through the neighborhoods, and others will go through the park. In the course of a year, there will be those who always make the same decision when faced with this circumstance, and there will be those who pick one route one day and another route the next time, and so on.

This puts us in an interesting position. Now we have two things we could study. The first is the similarity of the physical paths. The second is the distribution of personal choices when it's reasonable to consider taking an alternate path. How does the aggregate of the personal choices relate to the degree of similarity between the physical paths? Do people tend to make good choices with regard to directional similarity or bad choices? Are there factors beyond directional similarity which people take into account when making such choices?

Questions like these might show why an awareness of directional similarity might be interesting, but we are getting a bit off topic. Let's turn to another example of directional similarity.

Time was when anybody who wanted to tell time had to be able to read a clock which showed the time by the positions of its hands. In those days, it became customary to avoid the fastidiousness of reporting time in the manner of, "It's 8:53," and, instead, a loose approximation of the time would suffice, and we

would say, "It's ten to nine." Of course, you'd be just as right to say it was 5 till, and sometimes you could even get away with stretching it, so 8:53 could be reported as 'a quarter till' or even 'just about 9.' Nobody ever laid out the rules for this (or if they did, people did not adopt the practice because they had read the rules), but there was a general sense of what was an acceptable approximation, and what was just crazy. To say that 8:53 was 'half-past' or 'noon' or 'a quarter to 3' would have led people to think you weren't playing with a full deck, but, oddly, if you *always* reported the exact time—calling 8:53 '8:53' and 11:27 '11:27' and so forth—you would have run a similar risk of having people think you weren't quite right in the head.[2] Something about the positions of the hands on the face of a clock encouraged these approximations, and doing the approximations correctly counted as a kind of assurance in regard to both your sanity and your humanity.

The key, of course, was directional similarity. Someone with a well-honed sense of directional similarity would immediately know if the hand positions were 'close enough' to justify rounding the reading up or down to one of the traditional reference points.[3] And people just fell into this habit naturally. Directional similarity is that easy; for most people it doesn't require a special effort of thought in order to recognize it.

A third example is found on a compass. The face of a compass is marked with the four principle directions: north (N), south (S) east (E) and west (W). Many compasses will have labels at the half-way points, NE, SE, SW and NW. Some compass manufacturers get quite taken away with this system, and supply all kinds of helpful intermediary directions. The interesting point to realize is that this system of naming clearly indicates that it is perfectly natural for us to regard the directions which fall between N and E, as being a mix of *both* of these directions. By way of contrast, my toy robot (if he had had the power of speech) would have complained bitterly, "You humans are crazy! That direction is neither north nor east! How can you call it northeast?"

That robot was too rigidly fixed in his conception of direction: for him, there was only one north, and everything else was 'not north.' We humans don't see it that way. There's north, but there's also northwards, and 'northwards' can encom-

[2] The digital clock, therefore, has done us all the disservice of making it that much harder to detect the loonies in our midst. No wonder the world's become such a crazy place since the 1960s.

[3] In general, the reference points were limited to a half-hour range. Only when something special was at stake, for example, in the countdown to midnight on New Year's Eve, would somebody be able to get away with saying something like "it's forty minutes till."

pass just about any mix of north with east or west, from 99.9…9% east with 0.0…1% north to 99.9…9% west with 0.0…1% north. In fact, given enough whiskey, probably about half of us would be happy to say, “Tell you what, why don’t we just include pure East and pure West into what we’re calling ‘northwards’ and be done with it. After all, what we're really trying to avoid is getting mixed up with 'southwards', not East and West.”

By bringing percentages into the conversation, we have quietly crossed the line into mathematics. Now, that’s scary country for some folks, so were not going to rush off into polynomials and quadratic equations and what-all. Instead, we’re going to take the artistic route and draw some pictures, because, like Alice said, “What good is a book without pictures?”